Whitby Town Schools Partnership

Methods for written calculation

Agreed on 13th March 2014
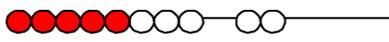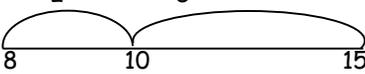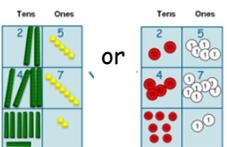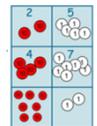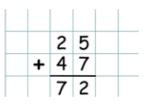
by
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How to... methods for written addition

The methods outlined below identify progression from introductory concepts to the end of Key Stage Two expectations (as described in the 2014 National Curriculum) When teaching written methods, the emphasis should be placed on <u>understanding</u> the concept/operation. Whilst initially this may not always be the quickest way to solve the problem, it is imperative children understand why they are working in this way and the reasoning behind the methods being used. In time, and once understanding is in place, children should be taught simpler, more efficient methods.

In the New Curriculum, a large emphasis is placed on using practical equipment for solving the four operations. Teachers should provide explicit opportunities to use such equipment when teaching and completing calculations.

Stage	Explanation	Example
1: Concrete Objects	Practical equipment, models and images should be used to provide opportunities for children to recognise and visualise units and values. Initially, recording of calculating should be modeled by adults in order to demonstrate what has been done. This recording should, of course, over time, be transferred to the child.	*How many bears?* one, two, three How many altogether? Let me show you what that looks like as a number sentence: 1 + 1 + 1 = 3 Can we write this any other way?
2: Number tracks	Number tracks can be used to explore the position and order of numbers. They develop the ability to visualise the number position with what comes before or after a given number. This should be taught progressively, from all units given, then perhaps remove the odd numbers, then only show every five etc. Think practically... what about rulers? Bead strings? 100 squares? Etc.	**8 + 7 = 15** **8 + 2 = 10**
3: Number lines	The empty number line helps to record the steps on the way to "jumping up" or calculating the total. The steps in addition are recorded on a number line. *(see example)* Seven is partitioned into 2 and 5; creating a number bond to 10 with the 8. Then, the five is added to the ten to reach the answer of 15. Depending on ages and stages, children's jumps will vary.	**8 + 7 = 15** +2 +5 8 10 15 **48 + 36 = 84** +6 +30 48 54 84
4. Partitioning	The next stage is to record mental methods using partitioning (breaking the number up) This involves finding all of the same value (e.g. Partition the tens, then units etc.) Adding the biggest digits first to form partial sums and then add answers of the partial sums. Think practically... what about Cuisenaire rods? Dienes? Place value cards? Etc.	**48 + 36 = 84** 40 + 30 = 70 8 + 6 = 14 70 + 14 = 84 / **143 + 89 = 232** 100 + 0 = 100 40 + 80 = 120 3 + 9 = 12 100 + 120 + 12 = 232
5. Practical equipment	Not all children would require this stage. However, using practical equipment as a route to written methods helps children to visualise the written algorithm and understand, using the practical tools, what is happening (see example of 25 + 47)	or leading to..
6. Column Method	Column addition is first introduced in the New Curriculum at Y3. Children should be taught to add vertically, units first. Carried digits are recorded below the line. Ensure that when explaining children use the words, 'carry ten' or 'carry one hundred' not 'carry one'.	48 +36 84 1 143 + 89 232 11 2345 + 1223 3568

How to… methods for written subtraction

The methods outlined below identify progression from introductory concepts to the end of Key Stage Two expectations (as described in the 2014 National Curriculum) When teaching written methods, the emphasis should be placed on <u>understanding</u> the concept/operation. Whilst initially this may not always be the quickest way to solve the problem, it is imperative children understand why they are working in this way and the reasoning behind the methods being used. In time, and once understanding is in place, children should be taught simpler, more efficient methods.

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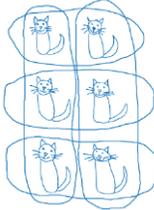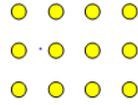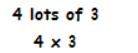
Stage	Explanation	Example
1: Concrete Objects	Practical equipment, models and images should be used to provide opportunities for children to recognise and visualise units and values. Initially, recording of calculating should be modeled by adults in order to demonstrate what has been done. This recording should, of course, over time, be transferred to the child. The 'difference between' is introduced through practical situations and images	*There are four children in the home corner. One leaves, how many are left now?* *What is the difference?*
2: Number tracks	Using a number track supports children to find how many items are left after some have been 'taken away'. Number tracks can be used to explore the position and order of numbers. They develop the ability to visualise the number position with what comes before or after a given number. This should be progressively taught. Think practically… what about rulers? Bead strings? 100 squares? Etc.	 How many are left when 2 are taken away from 10? count back one, two or three
3a: Number lines to COUNT BACK	**COUNTING BACK** is a useful strategy when the context of the problem results in their being less, e.g. Bill has 15 sweets and gives 7 to Jack, how many does he have left? As in addition, children need to be able to partition the jumps. A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. Depending on ages and stages, children's jumps will vary.	 Counting back from 74 to the nearest ten… etc.
3b. Number lines to COUNT ON	**COUNTING ON** is finding the difference from the smaller to the larger number. For example, counting up from 27 to 74 in steps (see example from COUNTING BACK) This is a useful method when the context asks for comparisons – How much longer? How much smaller? For example, Jill has knitted 27cm of her scarf and Alex has knitted 74cm. How much longer is Alex's scarf?	**74 – 27 =** Count on from 27 The 'jumps' should be added, either mentally or with jottings according to confidence, beginning with the largest number e.g. 40 + 4 + 3.
	For both counting back and counting on, the method of drawing the number line will remain the same – just the direction that is being travelled in will differ. After practice of both, examples like this will illustrate how children might choose when it is appropriate to count on or back. This also helps to reinforce addition and subtraction as inverses and the links between known number facts.	
5. Practical	Not all children would require this stage. However, using practical equipment as a route to written methods helps children to visualise the written algorithm and understand, using the practical tools, what is happening (Example: 72 - 47)	leading to…
6. Column Method	The "traditional method", column subtraction is introduced in the New Curriculum at Y3. Children should be taught to subtract and take from the next digit when needed. Ensure that when explaining children use the words, 'carry ten' or 'carry one hundred' not 'carry one'.	563 – 246 = 317

How to... methods for written multiplication

The methods outlined below identify progression from introductory concepts to the end of Key Stage Two expectations (as described in the 2014 National Curriculum) When teaching written methods, the emphasis should be placed on <u>understanding</u> the concept/operation. Whilst initially this may not always be the quickest way to solve the problem, it is imperative children understand why they are working in this way and the reasoning behind the methods being used. In time, and once understanding is in place, children should be taught simpler, more efficient methods.

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Stage	Explanation	Example
1: Repeated Addition	Repeated addition helps children to understand the concept of 'lots of', e.g. 3 lots of 3 could be written as 3 + 3 + 3 = 9 Think practically… On a bead string, children can count out three lots of 5s, then count the beads altogether. On a number line, children could count on in groups of 5.	3 lots of 5 using bead string: 5 + 5 + 5 = 15
2. Practical Equipment / Arrays	Practical equipment, models and images should be used to provide opportunities for children to recognise and visualise everyday arrays and repeated addition. For example, discussions with the number of eggs needed to fill a box. Children will also begin to draw their own pictures to recognise the nature of repeated addition (see example of cats) This also introduces the concept of multiplication being 'commutative' (the order of the digits will not impact upon the result, so 4 x 3 is the same as 3 x 4)	*How many cats are there?* *Three lots of two* *Two lots of three*
3. Arrays	It is important to be able to visualise multiplication as a rectangular array. This helps children develop their understanding of the commutative law, e.g. 3 x 4 = 4 x 3 The rectangular array allows the total to be found by repeated addition and the link can be made to the x sign and associated vocabulary (lots of, groups of etc.)	3 lots of 4 3 X 4 4 lots of 3 4 x 3
4. Grid method	Links between arrays and the grid method should be made clear to children by the use of models and visual representations. Children should be able to use practical equipment to visualise the link between the array and the grid. (see example) Once secure with the relationship between arrays and grid, children will use larger numbers to multiply. A grid is formed by partitioning the numbers into values and multiplying these smaller chunks.	4 x 13 10 3 40 12 Children should be shown how this model shows 4 x 13 but the calculation steps are simplified by partitioning the 13 into 10 and 3. This then becomes | ×	**10**	**3**
4	40	12 | 40 + 12 = 52 **TU x TU** 42 x 26 = | X	20	6
40	800	240
2	40	12 | 800 + 240 + 40 + 12 = 1092

5. Column Method (Long and Short)	The first step is to represent the method of recording in a column format, but showing the working. Draw attention to the links between the grid method. Children's understanding of place value is vital so they recognise when they are multiplying tens, hundreds etc. they record their answer in the correct columns. All carried units should be below the answer line.
	Short multiplication (HTU x U)
	Long multiplication (HTU X TU or bigger) *Specified to be taught in Y5 at the latest*
		342 × 7 becomes
		
		  3 4 2
		× 7
		2 3 9 4
		  2 1
		Answer: 2394
		
		124 × 26 becomes
		
		   1 2
		  1 2 4
		× 2 6
		 7 4 4
		2 4 8 0
		3 2 2 4
		 1 1
		Answer: 3224

How to... methods for written division

The methods outlined below identify progression from introductory concepts to the end of Key Stage Two expectations (as described in the 2014 National Curriculum) When teaching written methods, the emphasis should be placed on <u>understanding</u> the concept/operation. Whilst initially this may not always be the quickest way to solve the problem, it is imperative children understand why they are working in this way and the reasoning behind the methods being used. In time, and once understanding is in place, children should be taught simpler, more efficient methods.

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Stage	Explanation	Example
1. Sharing / Grouping	Division is introduced with real life objects and the idea of sharing or grouping. Practical examples should be offered to provide children with opportunities to physically group and/or share objects. SHARING should see children equally sharing/counting out the given value between the given number of groups. GROUPING should see the children equally grouping/splitting the given value into the given number of groups.	15 ÷ 3 **SHARING** **GROUPING**
2. Bead strings to introduce a number line	Practical equipment, models and images should be used to provide opportunities for children to recognise and visualise everyday division examples. Bead strings are a great tool to allow children to practically share a value into groups. This leads well onto the concept of a number line for division.	15 eggs are placed in baskets, with 3 in each basket. How many baskets are needed? Bead strings are used before making the link to a number line: 15 ÷ 3
3. Chunking on a number lines	Once the link has been established between sharing/grouping and bead strings with number lines, children should progress to chunk on a number line. This will involve counting up in jumps of the given number. Children should be encouraged to first find KFC (Known Number Facts) – see example	**81 ÷ 3 = 21** <u>KFC</u> **What facts do I know about the 3s?** 5 x 3 = 15 10 x 3 = 30 15 x 3 = 45 20 x 3 = 60 10 x 3 10 x 3 7 x 3 0 30 60 81 10 + 10 + 7 = 21
4. Short Division	For calculations where numbers with up to 4 digits are divided by a single digit number, children are expected to use short division.	**432 ÷ 5 = 86 r 2** Answer: 86 remainder 2
5. Long Division	For calculations where numbers of up to 4 digits are divided by a two digit number, children are expected to use long division.	**432 ÷ 15 = 28 r 12**